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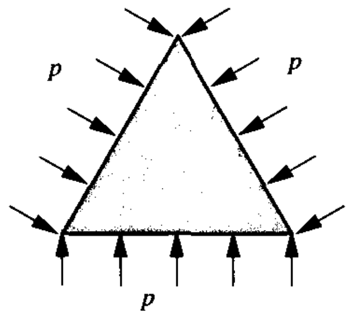
Tentamen
SOLID MECHANICS (NAVSM)
August 24 2006, 9–12 h

Question 1 In two dimensions, consider a regular triangular body subject to a uniform hydrostatic pressure p (as if submersed in a fluid).



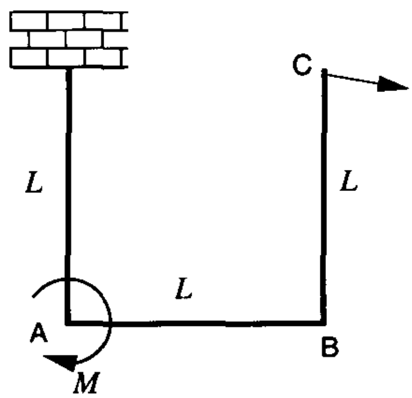
For a finite element analysis, the body is meshed with triangular elements, so that each of the three boundary edges contains N nodes. These nodes are connected along each edge by $N-1$ elements, inside of which the displacements are interpolated by linear shape functions $\mathsf{N}^I(s)$, where s is a coordinate along an edge from one corner of the triangle to the other.

a. Write down the shape functions $\mathsf{N}^I(s)$.

b. Determine the work-equivalent nodal forces along each edge.

c. Verify if the resulting set of all nodal forces satisfies equilibrium. Can one prove if this is so for any shape of the body?

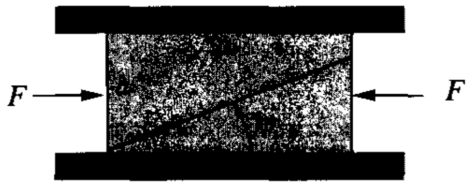
Question 2 A U-shaped pin with a total length of $3L$ is clamped at one end. When it is subjected to a bending moment M at the corner A, the end-point C will displace.



a. Express the displacement *vector* at point C in terms of M, L and the bending stiffness EI.

b. Alternatively, the moment may be applied at point B instead. Compute the displacement of C for this loading situation.

Question 3 Consider a block of material of dimensions $a \times 2a$ subjected to plane strain conditions perpendicular to the plane of the picture. The block is loaded by a horizontal compressive force F in the horizontal direction, which induces a uniform stress state when edge effects are neglected.



a. First consider the situation in the above figure where the block is being constrained against deformation in the vertical direction. There is no friction between the block and the two constraint platens. Determine the three-dimensional stress state.

b. Determine the stiffness in the direction of loading, assuming that the material remains elastic (and isotropic).

c. When the force becomes high enough, yield will start. What is the resolved shear stress on a plane inclined at the angle φ as shown? What is the most favourable orientation for slip to occur?

d. Answer the same question when the block is free to deform in the vertical direction.

Question 4 Check if the following Cartesian stress and strain components are physically allowable:

$$[\sigma_{ij}] = \begin{bmatrix} ayz & -dyz & dzy \\ -dyz & bxz & dz(y-x) \\ dzy & -d(x-y)z & cxy \end{bmatrix}, \quad [\varepsilon_{ij}] = \begin{bmatrix} x^2z & bxyz & 0 \\ 0 & az(x^2+y^2) & 0 \\ bxyz & 0 & cy^2 \end{bmatrix}.$$

Here, $\{x,y,z\}$ is short-hand for Cartesian coordinates $\{x_i\}$ $(i = 1,2,3)$, and a through d are constants.